RESEARCH DEPARTMENT

RING-TYPE TRANSMISSION-LINE NETWORKS

Report No. G-075

(1959/18)

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1. INTRODUCTION

At v.h.f. and higher frequencies increasing use is being made of filter networks employing closed rings of transmission line or waveguide. The theory developed here enables them to be related in a simple way to lumped-impedance networks. This not only provides a method of analysing well known arrangements, but is a useful tool in designing new arrangements based on a given lumped-impedance network. The utility of a transmission-line ring stems from the fact that all the impedance elements or terminals may be unbalanced, i.e. may have one connection at earth potential, so that special transformers, or multiple concentric feeders are not needed.

As a starting point, a simple transmission-line equivalent of a ring of lumped impedances is deduced; this is strictly valid at a single frequency only. The use of this equivalent in building up useful networks is discussed by giving examples. A simple method is then derived for calculating how the performance at any frequency differs from that at the design frequency. Finally, some extensions of the theory are considered and further examples of transmission-line rings are given.

2. THE RING-STAR TRANSFORMATION

The transformation to be described makes use of a simple property of a quarter-wave line. Fig. 1 illustrates a coaxial transmission line XY of characteristic impedance Z_0 and defines the input and output voltages, v_x, v_y and currents i_x, i_y. When the electrical length is $\pi/2$ radians we have, for a lossless line, the general relations

$$\begin{aligned} v_x &= jZ_0 i_y \\ v_y &= -jZ_0 i_x \end{aligned} \qquad (1)$$

i.e. the voltage at one end is controlled entirely by the current at the other end under all conditions of load.

If we have a ring of lumped impedances, A, B and C, as shown in Fig. 2 (a), the essential conditions which must hold are the Kirchoff rules:

(i) the currents in A, B and C are identical, and
(ii) the sum of the voltages across A, B and C is zero

For the purpose of this discussion an impedance referred to as A, etc., may be any two-terminal network which may include generators; at least one generator must, of course, be present in the system of impedances for a finite current to exist. Consider now three quarter-wave lines connected in a star as shown in Fig. 2(b). The outer ends

are terminated by impedances A, B and C respectively, while at the inner ends the three lines are connected together in parallel. It should be noted that no impedance or generator exists at the central junction, so that, at the inner ends of the three lines

(i) the voltage is the same, and
(ii) the sum of the outward currents is zero

If equations (1) are now applied to give the conditions at the outer ends, we arrive at the same conditions as those applying to the ring of impedances in Fig. 2(a). The arrangement of Fig. 2(b), which is represented diagrammatically in Fig. 2(c), is thus an equivalent of the ring of impedances. Although the use of two quarter-wave lines as a means of introducing a series impedance in a coaxial-line network has been described, for example, by Cork,[1] the extension of the principle in the form given here does not appear to be widely known.

This type of transformation may be extended in several ways. First, we may have any number of impedances in the ring with the corresponding number of radial lines in the star; Fig. 3 illustrates this extension for four impedances. Second, any length of line may be increased by an integral number of wavelengths, without affecting the equivalent, and by an odd multiple of a half-wavelength provided the phase reversal is taken into account. Precise equivalence is, however, dependent on zero loss in the lines and on their electrical length being exactly one quarter-wavelength (or an odd multiple thereof) at the frequency under consideration. Some other variants of the transformation are given later in Section 5.

3. SOME SIMPLE TRANSMISSION-LINE NETWORKS

3.1. Ring of Four Quarter-Wave Lines

A simple transmission line network of interest is a ring of four quarter-wave lines as depicted in Fig. 4(a). A further quarter-wave line (of the same characteristic impedance, Z_o) may be added to the terminal to which P is connected but to preserve the equivalence this additional line must be terminated by impedance P' where $P' = Z_o^2/P$; this is then seen as impedance P at the original terminal. If a similar modification is also made at the opposite corner, with $Q' = Z_o^2/Q$, we obtain two stars as shown in Fig. 4(b). The equivalent circuit of Fig. 4(c) may then be derived. This contains impedances P', S, T and S, T, Q' in two rings with S and T as common impedances.

An application of this example is a constant-impedance filter, such as a vestigial side-band filter,[2] shown in Fig. 5(a). The impedances Z are usually provided by two identical stub arrangements. The equivalent circuit of Fig. 5(c) shows that these impedances require to be high at pass frequencies and low at stop frequencies. For this circuit it is well known that the input impedance is R at all frequencies provided the load remains constant at R.

Another case of interest is a variation of the example of Fig. 4 in which there is a reversal in the polarity of connection at one point as indicated in Fig. 6(a). This is normally achieved in practice by inserting an extra half-wavelength

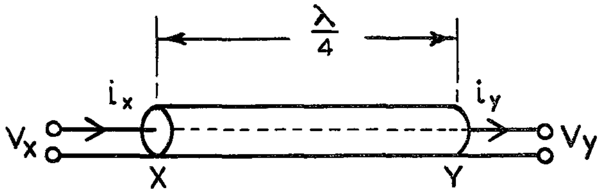

Fig. I - Quarter-wave transmission line

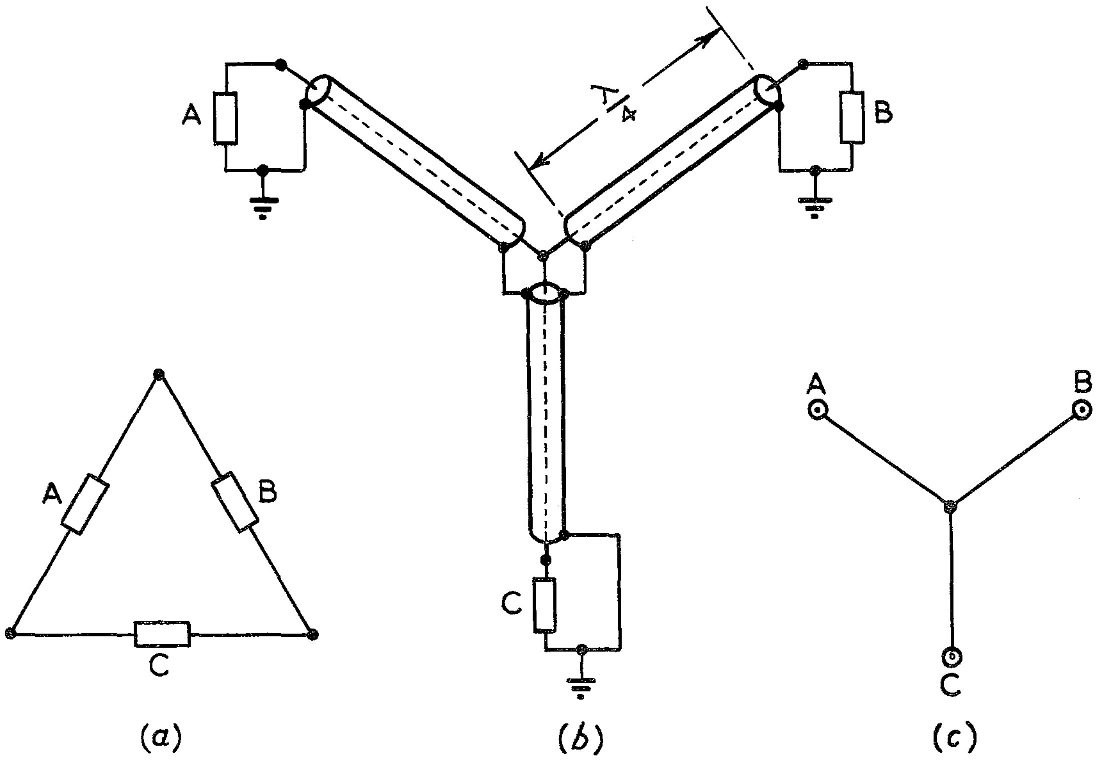

Fig. 2 - (a) Ring of impedances
(b) Star of quarter-wave lines
(c) Diagrammatic representation of (b)

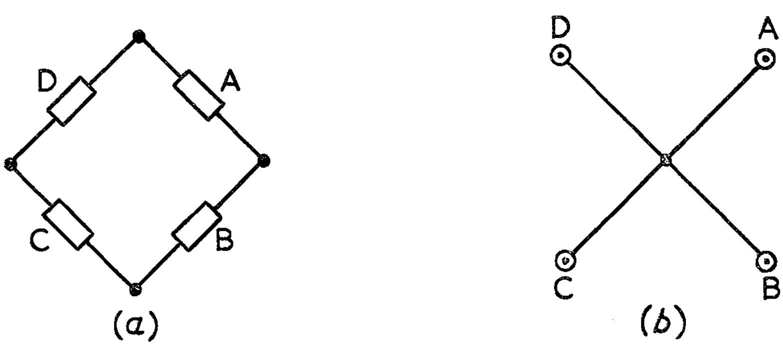

Fig. 3 - (a) Ring of impedances
(b) Star of quarter-wave lines

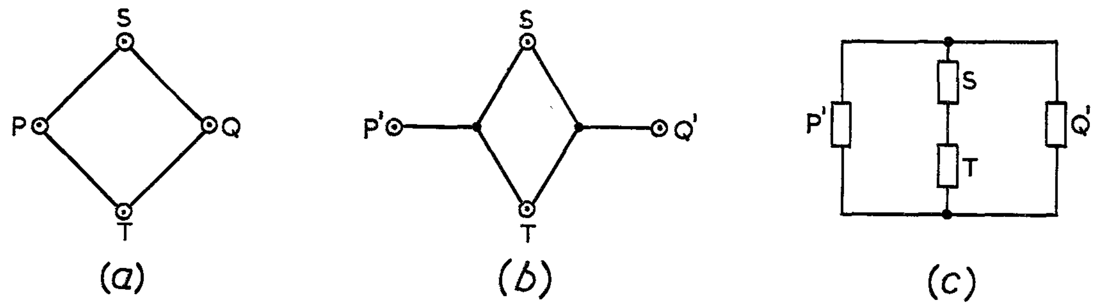

Fig. 4 - (a) Ring of four lines, characteristic impedance Z_0
(b) Ring modified to give two line-stars
($P' = Z_0^2/P$, $Q' = Z_0^2/Q$)
(c) Lumped-impedance equivalent circuit

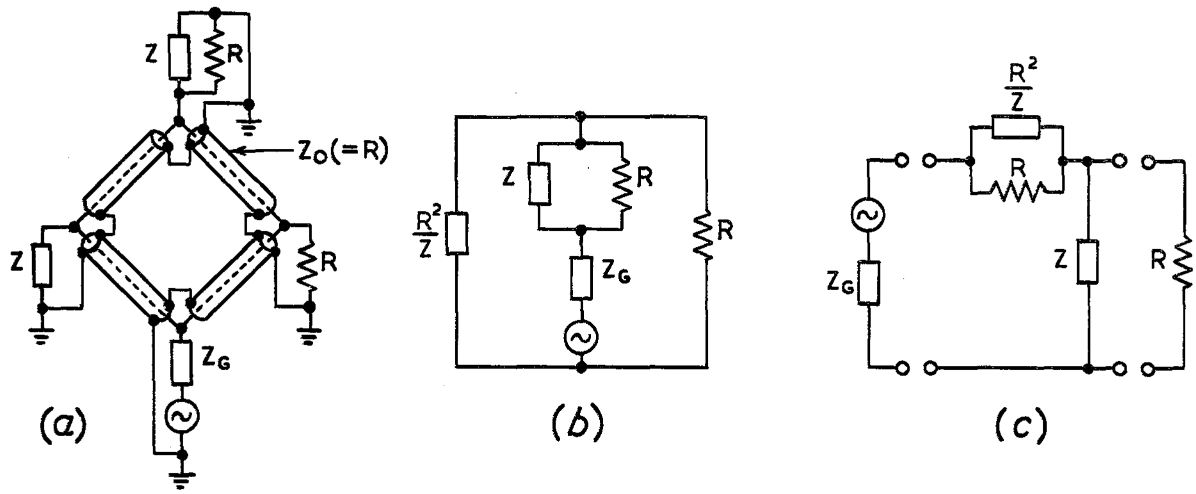

Fig. 5 - (a) Constant-impedance filter
(b) Equivalent circuit from Fig. 4
(c) Equivalent circuit redrawn as a filter section

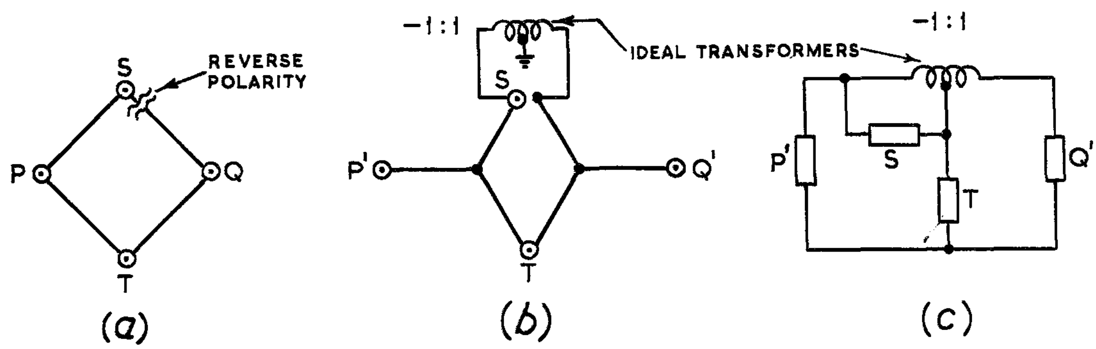

Fig. 6 - (a) Example as in Fig. 4, but with phase reversal
(b) Modification with an ideal transformer
(c) Equivalent circuit (hybrid transformer circuit)

of transmission line, though for transmission lines other than coaxial lines there may be a more direct method. The equivalent circuit is derived as before, but an ideal transformer is used in Fig. 6(b) and (c) to represent the phase reversal. Fig. 6(a) is known as a hybrid ring (or in its waveguide form as a "rat-race") while the equivalent in Fig. 6(c) is a well-known hybrid transformer circuit.

In Fig. 6(c), S and T are decoupled from one another if $P' = Q'$ or $P = Q$; also they are presented with an impedance $\frac{1}{2}P'$ or $Z_o^2/2P$. Similarly, P and Q are decoupled for $S = T$. The hybrid of Fig. 6(a) will thus work and be properly matched with all terminals terminated by a resistance $R = Z_o/\sqrt{2}$, but there are other possibilities, e.g. one pair of terminals matched to Z_o and the other pair terminated by $\frac{1}{2}Z_o$. It is worth noting that the hybrid transformer circuit of Fig. 6(c) is the equivalent of connecting between terminals P' and Q' a symmetrical lattice network having $2S$ and $2T$ as the respective impedances of the direct and cross-over arms.[3, 4]

3.2. Transmission-Line Equivalents of a Bridge Circuit

Fig. 7(a) shows the general case of a bridge circuit. By the use of the equivalents of Figs. 2 and 3 the complete equivalent line network shown in Fig. 7(b) may be derived. For many applications this may be simplified to the network of Fig. 7(c) by removing the quarter-wave lines which feed terminals P, Q and B. Whereas Fig. 7(b) is a complete equivalent using impedances identical to those in Fig. 7 (a), the simplified form must have inverse impedances P', Q' and B' at the points shown, where $P' = Z_o^2/P$, etc.

Fig. 8 illustrates a practical application.[5] A constant-impedance bridge network of the type shown in Fig. 8(a) is of advantage in combining the outputs of two transmitters, T1, T2 to feed a common load R_A. The magnitude of the impedance Z is made small over the band of frequencies required from T1 and large over the band from T2. By this means relatively little power is wasted in the "balancing" load R_c. A high degree of isolation between the transmitters can be maintained by designing the bridge circuit to be balanced at all frequencies. This means making $R_A = R_c$ and maintaining the impedance R_c^2/Z at its correct value as closely as possible over the frequencies of interest.

At v.h.f. and higher frequencies, where coaxial lines are desirable for impedance stubs as well as for transmitter and load terminals, the equivalence of Fig. 7 offers a convenient arrangement, as shown in Fig. 8(b). Advantage has been taken of the need for a reciprocal impedance B', since if $Z_o = R_c$ the requirements for R_c^2/Z in the original circuit may be met by making B' equal to Z; the two frequency-selective elements in the final arrangement may therefore be identical. The length of coaxial feeder at a transmitter terminal is arbitrary and impedance inversion there is of no consequence.

Another coaxial equivalent of a bridge circuit exists; it may be derived if the circuit is first re-arranged as in Fig. 9(a). The full equivalent is shown in Fig. 9(b), but in applying the transformation of Fig. 2 to the three impedance rings one subtlety arises as follows. Any convention for the positive polarity of a current in the three rings will conflict for at least one of the impedances. In the example of Fig. 9(a) the convention shown by the arrows conflicts for the impedance A. A similar difficulty occurs with the voltages. However, by a reversal of

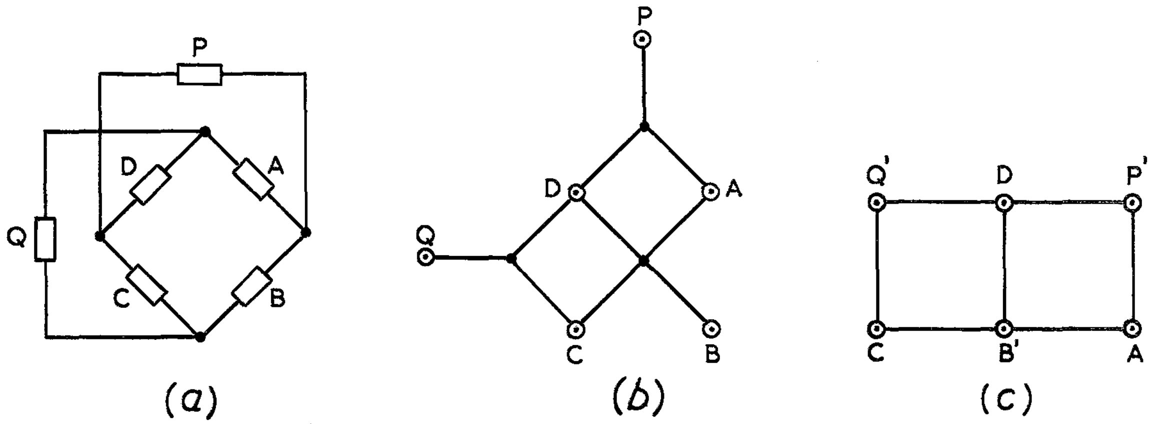

Fig. 7 - (a) Lumped-impedance bridge circuit
(b) Equivalent line network
(c) Simplified line network

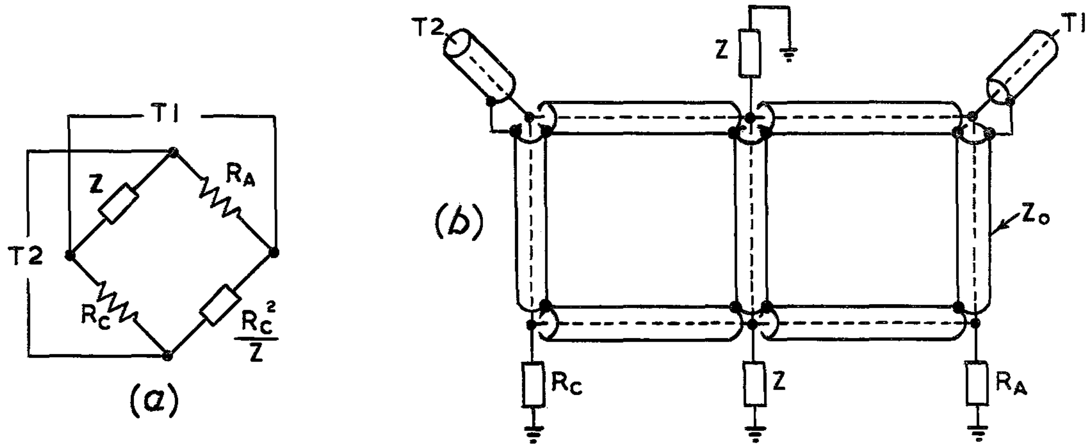

Fig. 8 - (a) Transmitter combining network
(b) Practical equivalent based on Fig. 7

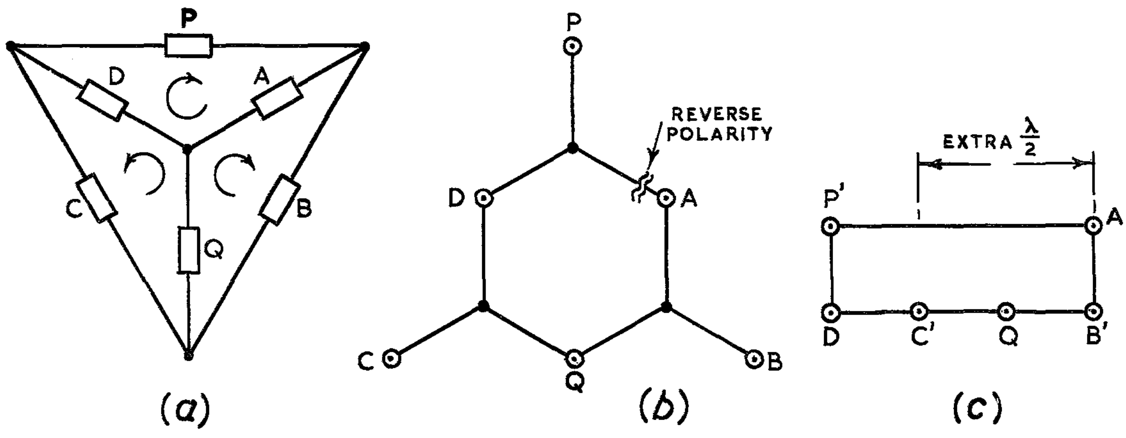

Fig. 9 - (a) Bridge circuit re-arranged
(b) Equivalent line network
(c) Simplified line network

polarity of connection of A into *one* of the two line-stars to which it is common, the convention indicated may be followed without any inconsistency arising. A practical form for coaxial line is shown in Fig. 9(c), the phase reversal being achieved by an extra half-wavelength of line. This arrangement is used as a transmitter combining network,[6] in much the same way as the previous example.

4. EFFECT OF FREQUENCY CHANGES

When the frequency departs from that at which the lengths of line are exact odd multiples of a quarter-wavelength, the performance will differ from that of the equivalent circuit. The equivalent for an arbitrary length of line, as shown in Fig. 10, is then most useful. An electrical length θ of lossless transmission line of impedance Z_o may be replaced by a quarter-wavelength of line of impedance Z_o' terminated at each end by a reactive impedance jX where

$$Z_o' = \pm Z_o \sin\theta$$
$$X = Z_o \tan\theta$$

These equations may be derived by equating the input impedances of the arrangements in Fig. 10(a) and (b) first when short-circuited at the other end and then when terminated by an impedance Z_o. The alternative sign in the first equation allows a positive value of Z_o' to be selected in all cases, though strictly Z_o' is negative if $\sin\theta$ is negative, indicating that a cross-over in the line of Fig. 10(b) is then needed for it to be an equivalent in all respects. Because the effect of line loss has been neglected the accuracy in practice will not be good if θ approaches π (i.e. if the line length approaches one half-wavelength) but will be very good for the case required here with θ near to $\pi/2$.

We can consider, for example, the consequence of a frequency rise of a few percent (fraction δ) above the design frequency. Then

$$\theta = \frac{\pi}{2}(1 + \delta) ,$$

and we may take

$$Z_o' = Z_o \cos\frac{\pi\delta}{2} \simeq Z_o$$

while jX becomes equivalent to a capacitance C given by

$$\omega C = (\tan\frac{\pi\delta}{2}) / Z_o \simeq \frac{\pi\delta}{2Z_o} .$$

The ring circuits discussed are therefore affected in the same way as if capacitances given by this expression were added at the end of each section of line. The sum of the capacitances associated with adjacent sections appears directly across each terminal impedance. In the case of a three-quarter wave line a capacitance C' given by

$$\omega C' = (\tan\frac{3\pi\delta}{2}) / Z_o \simeq 3\pi\delta/2Z_o$$

would have to be allowed for at each end. For frequency variations exceeding a few per cent, it may also be necessary to consider the consequence of changing the characteristic impedance from Z_o to Z_o' as required to simulate exactly the change in electrical length of the lines.

As an example of the effect of a small frequency variation it is interesting to discuss the networks of Figs. 7(c) and 9(c). We may take as a "unit" of mismatch

at the terminals the additional susceptance, $\pi\delta/2Z_o$, required at the ends of a single quarter-wave line.

In Fig. 7(c) there will be three units across D and B' while the other terminals will have only two units. In Fig. 9(c) there will be a total of four units across P' and A and of two units across each of the remaining terminals. The importance of these terminal mismatches depends on the application. Taking the one given in Fig. 8, we observe from Fig. 7(a) that the requirement for a bridge balance, i.e. isolation of P and Q, is that $AC = BD$. In the network of Fig. 7(c), two of the terminals have impedances of $B' = Z_o^2/B$ and D. If $B' = D$, then $BD = Z_o^2$; moreover, if B' and D are modified by the parallel addition of two identical impedances, their equality is unaffected, and BD remains equal to Z_o^2. But the other two terminals which can affect the balance have impedances A and C directly across them. Supposing $AC = Z_o^2$ initially, modification of A and C by equal impedances in parallel will alter the value of the product AC, and will thus cause a first-order disturbance of bridge balance with change of frequency.

In Fig. 9(c), on the other hand, the pairs of terminal impedances are B', D and A, C'. If the extra half-wavelength were replaced by an ideal phase reverser, the impedances required to simulate a frequency change would be equal at all four of these terminals; the bridge balance condition $AC = BD$ would then be unaffected since both of the conditions $B' = D$ and $A = C'$ could be maintained. With the extra half-wavelength used in practice the net result of a frequency variation is a degree of unbalance which is about one half of that experienced with the network of Fig. 7(a).

So far we have considered only the bridge balance. The impedance presented to the input terminals may also be important, particularly if more than one network is used in tandem. In a practical application of the type illustrated in Fig. 8, the impedance is of special interest when Z is very large or very small. In this case, with B' and D equal to Z, the network of Fig. 7(c) affords first-order impedance compensation when either of the input terminals P and Q supply power to A, whereas the network of Fig. 9(c) gives first-order impedance compensation only for the case of P supplying power to A. This may be demonstrated from the fact that in the former network we have P', Q' and A at the terminals whereas in the latter we have P', Q and A.

5. FURTHER EXTENSIONS OF THE TRANSFORMATION

The equivalents of the type discussed in Section 2 may be adapted to cover two cases not considered so far. The first is that of unequal transmission-line impedances and the second is that in which transmission-line junctions with a series instead of a parallel connection are practicable as, for example, with waveguides.

5.1. Lines of Unequal Impedance

We consider three quarter-wave lines with characteristic impedances aZ_o, bZ_o and cZ_o respectively forming a star as in Fig. 11(a), being connected in parallel at one end and terminated by the respective impedances A, B and C at the other. We may then derive the equivalents of Fig. 11(b) and (c), based on a more general application of equations (1). The transforming effect of the different lines may be

allowed for either by applying a factor, $1/a^2$ etc. to the impedance values as in Fig. 11(b), or by including transformers of turns ratio $a{:}1$, etc. as in Fig. 11(c).

An interesting example of an unequal-impedance line network is shown in Fig. 12(a). As in the example of Fig. 4 we add quarter-wave lines to an opposite pair of terminals as in Fig. 12(b), with $P' = Z_o^2/P$ and $T' = Z_o^2/T$. The equivalent shown in Fig. 12(c) must now include transformers to allow for the reduced characteristic impedance, $Z_o/\sqrt{2}$, of the two sections shown by thick lines. By halving the impedances in the right-hand ring one transformer may be eliminated; the remaining one must then have a 2:1 turns ratio as shown in Fig. 12(d). A simple re-arrangement, using an auto-transformer, finally leads to the hybrid-coil circuit of Fig. 12(e). This shows that the arrangement is similar to the example of Fig. 6, but it should be noted that the balance requirements of the transmission-line networks are different. Thus the terminals S and T, which are on adjacent corners of the network of Fig. 12(a), are decoupled if $P' = Q$, i.e. if $PQ = Z_o^2$. Given this condition, the impedance presented to S equals Q, and that presented to T equals P. The usual mode of operation will be to make S, T, P and Q all equal to Z_o. A dashed symbol in the equivalent circuits may be taken to indicate that the impedance is seen as if through a quarter-wave line. Since P' and Q appear in Fig. 12(e) a signal applied to S in Fig. 12(a) will appear as two signals at P and Q which are in phase quadrature with one another, whereas in Fig. 6(a) they would be in phase with one another.

A further difference between the hybrid networks of Figs. 6 and 12 is the effect of a change of frequency. It was shown in Section 4 that a small variation of frequency from that for which the equivalent is derived is approximately equivalent to a reactance placed across each junction. If we suppose an ideal phase inverter is used in Fig. 6(a), then a frequency change is effectively represented by placing equal reactances at all four corners of the network, and the balance condition $P = Q$ is maintained. In the case of Fig. 12(a), on the other hand, the balance condition $PQ = Z_o^2$ is not preserved. In practice the use of a half-wave line as a phase reverser in Fig. 6(a) will also cause a first-order disturbance of the balance condition with frequency, though a net advantage in regard to balance remains in spite of this.

There is a disadvantage in regard to impedance matching when the half-wave line is used in the arrangement of Fig. 6(a), whereas the alternative of Fig. 12(a) has no first-order disturbance of the matching with frequency.

5.2. Line Networks with a Series Junction

In some cases, e.g. with waveguide transmission lines, it may be easier to arrange a junction in which the transmission line terminals are effectively connected in series with one another instead of in parallel. If one is restricted to series junctions alone, then the situation in regard to the relative difficulty of connecting impedance in series or in parallel is reversed as compared with coaxial-line networks. Thus we must now regard a star of quarter-wave transmission-lines as a means of connecting impedances in parallel as shown diagrammatically in Fig. 13. The equivalence of (a) and (b) is seen from the fact that at the inner end of the transmission lines the currents are equal and the voltage sum is zero. This ensures the inverse relation at A, B and C, namely equal voltages and a current sum of zero.

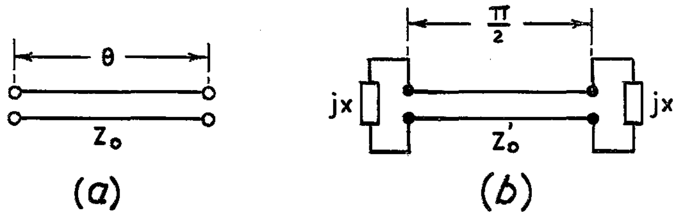

Fig. 10 - (a) Arbitrary length of line
(b) Equivalent using a quarter-wavelength line

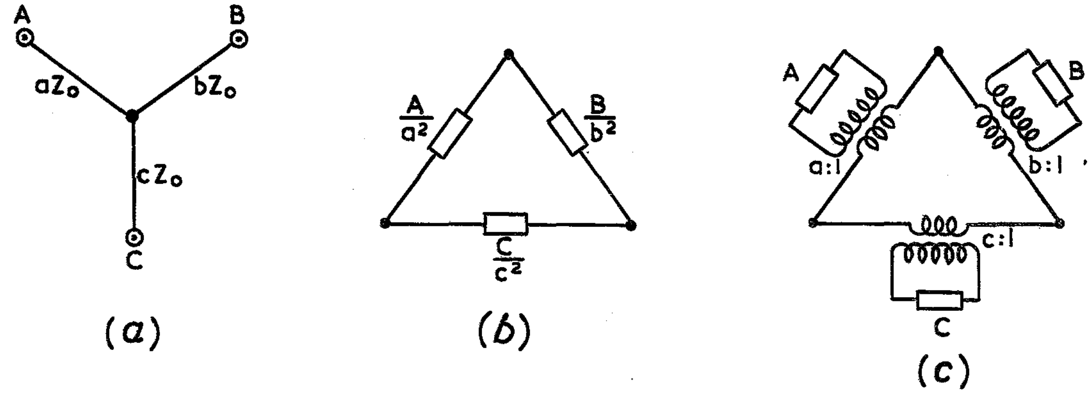

Fig. 11 - Two equivalents of star of lines of unequal impedance

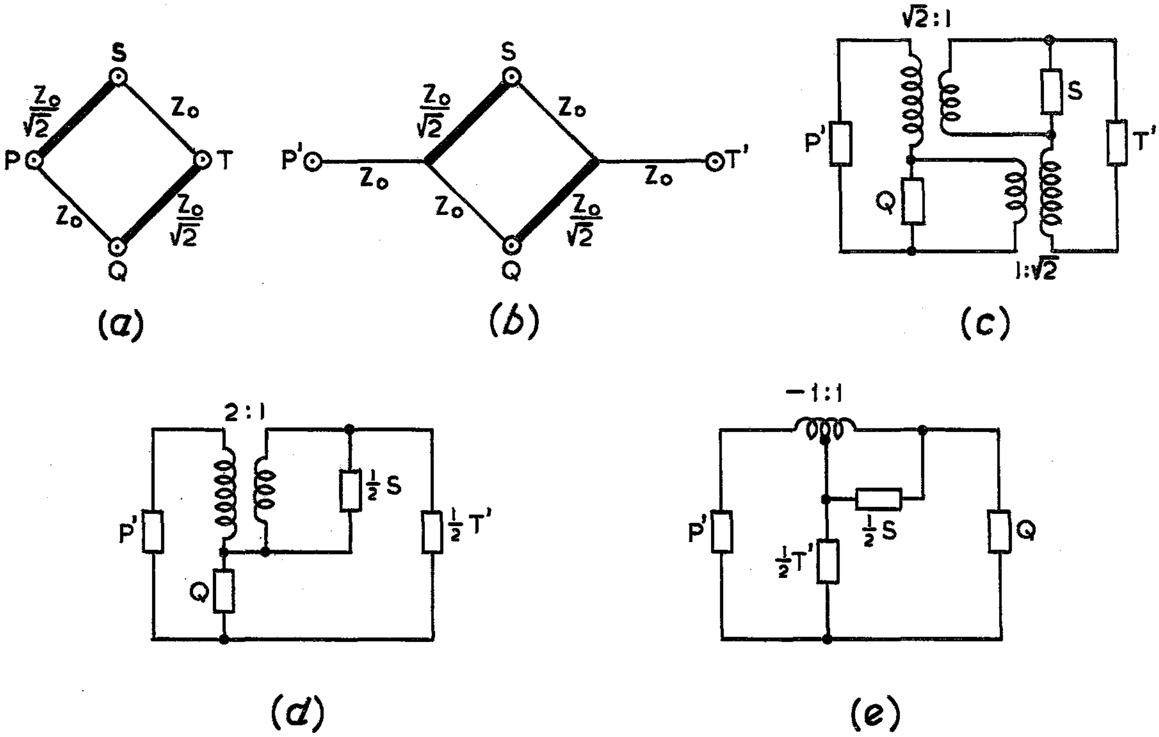

Fig. 12 - Network giving two signals in phase quadrature

We take as an example of deriving a series-line network the circuit of Fig. 7(a), which is repeated in Fig. 14(a). The separate rings of impedances have been indicated by thick lines in Fig. 14(b), and have been drawn spaced apart. Transmission line stars of the type shown in Fig. 13(a) have then been added in order to common (i.e. connect in parallel) the groups of terminals marked a, c or d. Finally the arrangement of Fig. 14(c), with $D' = Z_o^2/D$ etc., where Z_o is the characteristic line impedance, may be derived by analogy with Fig. 7.

It turns out in this example, and can be verified for the other examples, that series-junction line networks similar in form to the parallel-junction line networks result from a given lumped-impedance network, with the provision that where an impedance appears inverted (as indicated by dashed symbols) in one network it appears unmodified in the other.

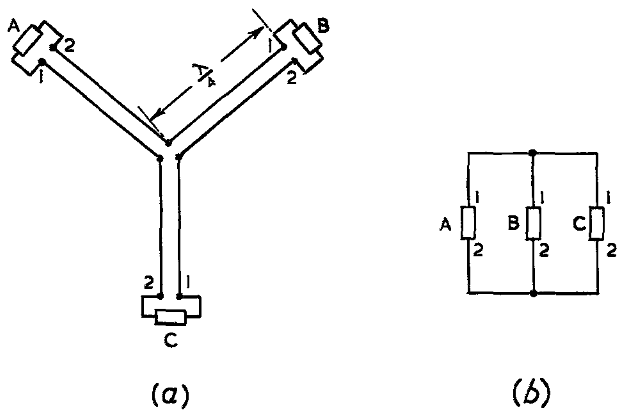

Fig. 13 - (a) Line-star with a series junction
(b) Equivalent circuit

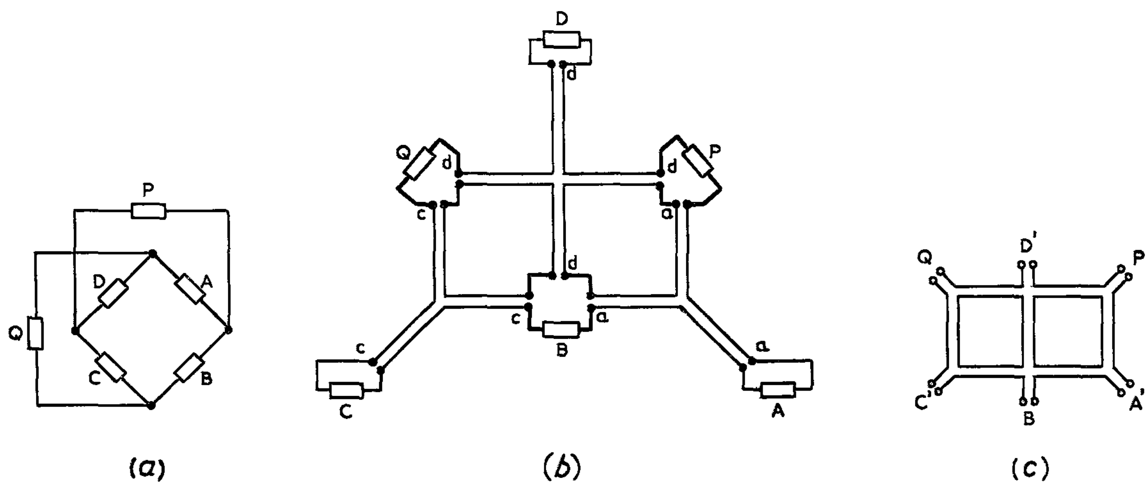

Fig. 14 - (a) Lumped-impedance bridge circuit
(b) Equivalent line network
(c) Simplified line network

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